WAVE PROCESSES IN A NONISOTHERMIC PLASMA

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The "coarse"-particle method [1-4] is used in this paper to solve the problem of decay of any ion-density rupture when the electron temperature is significantly higher than the ion temperature (in the calculations the ion temperature is assumed to be zero). As in [5-12] a model is chosen in which only the ionic-component motion is considered, and the electron density is described by the Boltzmann distribution. The necessity of using the coarse-particle method for solving the kinetic equations arises due to the fact that the hydrodynamic description of ion-acoustic waves in a nonisothermic plasma with $T_i = 0$ is valid only for waves of comparatively small amplitude:

$$\varphi_{\max} < \varphi_* = 1.26\ T_e / e, \qquad U < 1.58\ (T_e / m_i)^{1/2}$$

For large amplitudes and velocities the regular-solution wave structure is destroyed and a multicurrent flow is generated [5].

We start with the system of equations

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \frac{e}{m_i}\frac{\partial f}{\partial u}\frac{\partial \varphi}{\partial x} = 0 \tag{1}$$

$$\frac{\partial \varphi}{\partial x^2} = 4\pi e\left[n_0 \exp\left(\frac{e\varphi}{T}\right) - \int_{-\infty}^{\infty} f du\right] \tag{2}$$

where f is the ion distribution function, u is the ion velocity, φ is the potential, T is the electron temperature, and n_0 is the unperturbed plasma density.

The characteristic equations for the kinetic equation (1) are the equations of motion of ionic layers

$$\frac{du_j}{dt} = -\frac{\partial \varphi}{\partial x}, \qquad \frac{dx_j}{dt} = u_j \tag{3}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \exp(\varphi) - \rho \tag{4}$$

Here j is the particle (layer) number, ρ is the ion density, the velocity u is measured in units of ion-acoustic velocity $(T/m_i)^{1/2}$, the coordinate x is in Debye radii $D = (T/4\pi n_0 e^2)^{1/2}$, and the potential in units of T/e.

The algorithm of solving Eqs. (3), (4) is discussed in detail in [11]. In all calculations we used 1000 particles, and the length of the spatial interval was 150 D. The calculation time of a typical variant up to a time $t = 25\omega_{0i}^{-1}$ consisted of 15 min on a BÉSM-6 computer.

Consider the evolution of an initial ion-density rupture (a step) given by

$$\rho(x, 0) = \begin{cases} C = \text{const} & \text{for } 0 \leqslant x \leqslant x_0 \\ 1 + (C-1)\exp(-(x - x_0)^2 / l^2) & \text{for } x \geqslant x_0 \end{cases} \tag{5}$$

$$u_j(x, 0) = 0$$

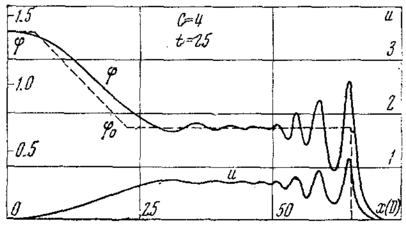

Fig. 1

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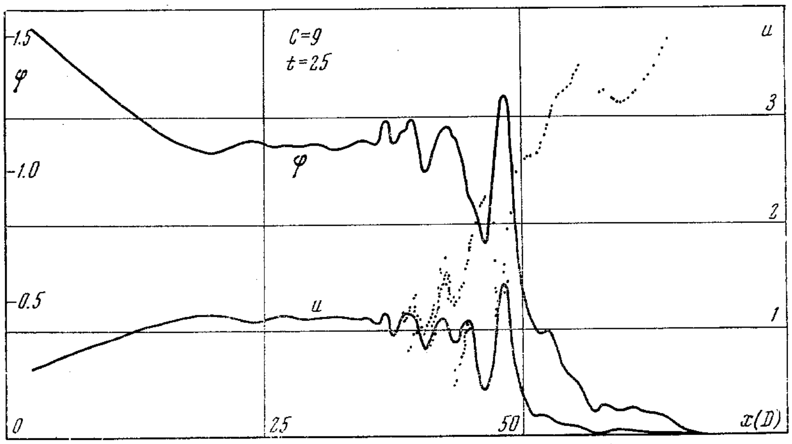

Fig. 2

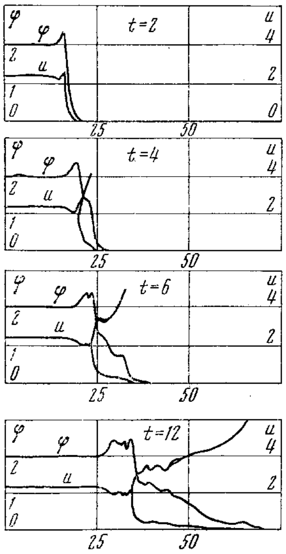

Fig. 3

For a relatively small density drop ($C \leqslant 4$) the step decay leads, as could be expected, to the formation of a laminar shock wave travelling right and a dilatation wave travelling left. The shock wave has an oscillating leading edge due to dispersion at the charge separation, which is accompanied by a portion with a predominantly constant amplitude $\bar{\varphi}$. We notice that the presence of this portion is related to a nonstationary process; for $t \to \infty$ the whole region should be filled by oscillators. The leading solution amplitude increases to some value $\varphi_{max} < \varphi_*$; front reversal and particle interchange do not occur. The results of these calculations were compared with the accurate solution of the rupture problem in dissipationless gas dynamics with an adiabatic index $\gamma = 1$, which corresponds to neglecting dispersion effects ($D \to 0$). The velocity of a gas-dynamic shock wave M_0 is determined from the transcendental equation

$$M_0^2 = C \exp[-(M_0 - M_0^{-1})] \tag{6}$$

and the values of the density, potential, and gas velocity at the front equal

$$\bar{\rho} = M_0^2, \quad \bar{\varphi}_0 = 2 \ln M_0, \quad \bar{u} = M_0 - M_0^{-1} \tag{7}$$

The potential amplitude $\bar{\varphi}$ agrees quite well with the quantity $\bar{\varphi}_0$ obtained from Eq. (7). The velocity of the oscillating shock wave M is somewhat higher than M_0 from (6), since it is determined by the amplitude φ_{max} of the leading solution. Figure 1 shows the results of calculating the evolution of a rupture with C = 4; the points correspond to the gas-dynamic potential profile.

For initial density drops $5 \leqslant C \leqslant 13$ ($0.8 \leqslant \bar{\varphi} < \varphi_*$) the leading soliton amplitude, unlike the previous case, increases to the value $\varphi_{max} \geqslant \varphi_*$, after which reversal occurs with the formation of a precursor and fast particles, reflected by the front. Calculations show that if $\bar{\varphi} < \varphi^*$, particle emission has a pulsating character, i.e., after the leading soliton amplitude increased to $\varphi_{max} \geqslant \varphi_*$, later increasing again due to the step energy, new particle emission takes place, etc. This fact has been noticed [5] for the evolution problem of symmetric compression. Figure 2 presents results of calculating rupture decay with C = 9.

An increase of the step amplitude C and of the corresponding potential amplitude $\bar{\varphi}$ ($C > 13, \bar{\varphi} > \varphi_*$) leads to the formation of a shock wave without regular oscillations, but with a sharp front between the fundamental part of the wave and the preceding part. At the same time continuous reflection of particles takes place. The formation of a precursor wave is accomplished by a retardation of the fundamental wave. Indeed, if a laminar shock wave always anticipates a gas dynamic profile, in the case considered the fundamental wave lags after it. For example, the velocity of the gas dynamic shock wave is $M_0 = 2.49$ at C = 50, and the wave velocity in the calculations is $M \approx 1.8$ for $D \neq 0$. The velocity of the precursor wave is, indeed, significantly higher and equals $M_p \approx 5.3$ in this case. In view of the fact that a further increase in the potential $\bar{\varphi}$ in the shock wave requires a significant increase in C (see below) and, correspondingly, in

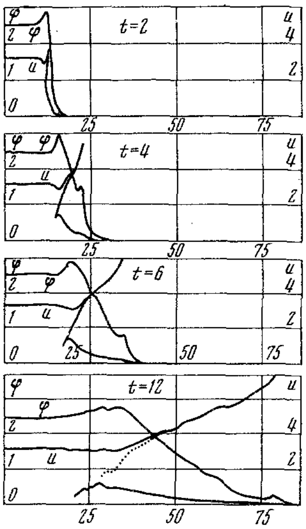

Fig. 4

the number of particles, for large amplitude calculations we used a modified model in which only the shock wave region was considered. The initial conditions

$\bar{C}$	50	80	300	1422	76880
M_0	2.49	2.73	3.48	4.48	7.39
$\bar{\varphi}$	1.82	2.01	2.49	3.00	4.00

are assigned in this case by the gas dynamic solutions and are of the form

$$\rho(x, 0) = \begin{cases} \bar{C} = \text{const} & \text{for } 0 \leqslant x \leqslant \bar{x}_0 \\ 1 + (\bar{C} - 1)\exp[-(x - x_0)^2 / l^2] & \text{for } x > \bar{x}_0 \end{cases} \tag{8}$$

$$\varphi(x, 0) = \ln \rho(x, 0) \tag{9}$$

$$u_j(x, 0) = [\rho(x_j, 0)]^{1/2} - [\rho(x_j, 0)]^{-1/2} \tag{10}$$

Figure 3 shows the potential profiles and particle velocities at various moments of time in case $\bar{\varphi} = 2$. Clearly seen is the formation of a current of fast particles and a precursor current, as well as a sharp front of the fundamental wave. With increasing potential amplitude $\bar{\varphi}$ the maximum potential value in the precursor wave φ_p increases (see below) and the difference between the fundamental wave and the precursor wave vanishes. This occurs for $\bar{\varphi} \approx 2.4$.

$\bar{\varphi}$	2	2.2	2.35
φ_p	1.25	1.5	2.0

The transition to amplitudes $\bar{\varphi} > 2.4$ causes a qualitative rearrangement in the nature of the process, instead of shock wave formation there occurs a continuous spread of the initial profile. This fact was observed experimentally [13] and was verified by calculations [11, 14, 15]. We notice that [11, 14] considered the evolution of the initial compression

$$\rho(x, 0) = 1 + C\exp[-(x - x_0)^2 / l^2]$$

and [15] the piston problem. Figure 4 shows profiles of the potential and particle velocities at various moments of time in case $\bar{\varphi} = 2.5$. Unlike the smaller-amplitude cases considered earlier, the particles can be sharply divided into two groups, fast and slow.

Thus, the study of evolution of any ion-density rupture in a nonisothermic plasma shows that, depending on the density ratio C or the potential amplitude $\bar{\varphi}$ there exist four qualitatively different cases:

1) $C < 5$, $\bar{\varphi} \leqslant 0.7$, a laminar oscillating shock wave;

2) $5 \leqslant C \leqslant 13$, $0.8 \leqslant \bar{\varphi} < \varphi_*$, reversal of the shock wave with pulsating particle reflection with formation of a precursor wave; the fundamental wave has a sharp leading edge;

3) $C > 13$, $\varphi_* < \bar{\varphi} < 2.4$, a shock wave with a sharp front between the fundamental and precursor wave, a potential profile without oscillations, continuous particle reflection;

4) $\bar{\varphi} > 2.4$, continuous rearrangement of the potential profile.

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